Siberian Federal University

Complex Analysis and Differential Equations Laboratory

Several Complex Variables

B.V. Shabat Centennial Conference

Program & Abstracts

September 11 – 15, 2017

Krasnoyarsk, Russia

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Timetable

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 – 9:30		Kang-Tae Kim	Timur Sadykov	Shinichi Tajima	Azimbay Sadullaev
9:30 – 10:00					
10:00 – 10:30		Stefan Nemirovsky	Irina Antipova	Alain Yger	Georgy Shabat
10:30 – 11:00					
11:00 – 11:30		Coffee	Vladimir Kulikov	Coffee	Coffee
11:30 – 12:00		Alexander Tumanov	Dmitry Pochekutov	Gabino González-Diez	Alexander Komlov
12:00 – 12:30			Lunch		
12:30 – 13:00		Nadezhda Chuesheva		Yury Eliyashev	Alekos Vidras
13:00 – 14:00		Lunch		Lunch	Lunch
14:00 – 14:30	Sergey Pinchuk				
14:30 – 15:00		Valery Beloshapka		Alain Hénaut	Victor Chueshev
15:00 – 15:30	Coffee				Armen Sergeev
15:30 – 16:00	Nikolay Kruzhilin	Coffee		Coffee	
16:00 – 16:30	Gerd Schmalz	Boris Bychkov		Sergey Smirnov	Coffee
16:30 – 17:00		Alexander Shlapunov			
17:00 – 17:30	Alexander Loboda	Gulmirza Khudayberganov		Sachiko Hamano	
17:30 – 18:00					

Schedule

Day I: Monday, September 11, 2017

- 13:50 – 14:00 *Opening*
- 14:00 – 15:00 Sergey Pinchuk : *Geometry of proper holomorphic mappings*
- Coffee
- 15:30 – 16:00 Nikolay Kruzhilin : *Proper mappings of Reinhardt domains*
- 16:00 – 17:00 Gerd Schmalz : *CR-manifolds and shear-free congruences of conformal Lorentzian manifolds*
- 17:00 – 18:00 Alexander Loboda : *On moduli spaces and examples of holomorphically homogeneous hypersurfaces in* $\mathbb{C}^3$
- 18:30 Buffet

Day II: Tuesday, September 12, 2017

- 9:00 – 10:00 Kang-Tae Kim : *On the generalization of Forelli's theorem*
- 10:00 – 11:00 Stefan Nemirovsky : *Continuation, approximation and topology in two complex variables*
- Coffee
- 11:30 – 12:30 Alexander Tumanov : *Symplectic non-squeezing in infinite dimension*
- 12:30 – 13:00 Nadezhda Chuesheva: *Some nonlinear equations of higher order*
- Lunch
- 14:30 – 15:30 Valery Beloshapka : *Analytic complexity: 10 years later*
- Coffee
- 16:00 – 16:30 Boris Bychkov : *Degrees of the strata of Hurwitz spaces*
- 16:30 – 17:00 Alexander Shlapunov : *Open mapping theorem for the Navier-Stokes equations in Hölder spaces*
- 17:00 – 18:00 Gulmirza Khudayberganov : *On integral formulas in the space of rectangular matrices*
- 18:00 – 19:00 Memorial meeting

Day III: Wednesday, September 13, 2017

- 9:00 – 10:00 Timur Sadykov : *Amoeba-shaped polyhedral complex of an algebraic hypersurface*
- 10:00 – 11:00 Irina Antipova : *Multidimensional Mellin transforms*
- 11:00 – 11:30 Vladimir Kulikov : *On convergence of multidimentional Mellin-Barnes integrals*
- 11:30 – 12:00 Dmitry Pochekutov: *Toric cycles in the complement of a complex curve in the 2-dimensional complex torus*
- Lunch
- 13:30 Excursion
- 19:00 Conference dinner

Day IV: Thursday, September 14, 2017

- 9:00 – 10:00 Shinichi Tajima : *A new effective algorithm for computing Grothendieck local residues*
- 10:00 – 11:00 Alain Yger : *Averaging procedures versus effectivity questions in the arithmetic setting*
- Coffee
- 11:30 – 12:30 Gabino González-Diez : *Universal covers of complex surfaces and arithmeticity*
- 12:30 – 13:00 Yury Eliyashev : *Supercurrents and generalized amoebas*
- Lunch
- 14:30 – 15:30 Alain Hénaut : *Implicit differential equations and planar web geometry*
- Coffee
- 16:00 – 17:00 Sergey Smirnov : *Darboux integrable systems*
- 17:00 – 18:00 Sachiko Hamano : *Variational formulas for hydrodynamic differentials and pseudoconvexity*

Day V: Friday, September 15, 2017

- 9:00 – 10:00 Azimbay Sadullaev : *Further developments of the pluripotential theory*
- 10:00 – 11:00 Georgy Shabat : *Meromorphic functions on Riemann surfaces with no more than 4 critical values*
- Coffee
- 11:30 – 12:30 Alexander Komlov : *Hermite-Padé approximants for meromorphic functions on a compact Riemann surface*
- 12:30 – 13:00 Alekos Vidras : *Cauchy-Fantappiè integral representation formula for a class of holomorphic functions in tubes*
- Lunch
- 14:30 – 15:00 Victor Chueshev : *Prym differentials with matrix characters on finite Riemann surfaces*
- 15:00 – 16:00 Armen Sergeev : *Adiabatic limit in Seiberg-Witten equations*
- Coffee

Titles and Abstracts

Day I: Monday, September 11, 2017

Sergey Pinchuk *Indiana University, USA*
Geometry of proper holomorphic mappings

Abstract: Since early 70s proper holomorphic mappings between domains in $\mathbb{C}^n (n > 1)$ and other geometric areas of Several Complex Variables have been intensively studied at the seminar directed by A.A. Gonchar, A.G. Vitushkin, and B.V. Shabat at Moscow State University. A number of important results, which essentially changed and developed the multidimensional complex analysis, are the product of the activity at the seminar.

In this talk I will discuss some geometric methods (such as the scaling method, the reflection principle, invariant metrics) and results of the theory of proper holomorphic mappings.

Nikolay Kruzhilin *Steklov Mathematical Institute, Russia*
Proper mappings of Reinhardt domains

Abstract: We describe classes of bounded two-dimensional Reinhardt domains that admit proper holomorphic mappings of multiplicity >1 on two-dimensional complex manifolds.

Gerd Schmalz *University of New England, Australia*
CR-manifolds and shear-free congruences of conformal Lorentzian manifolds

Abstract: A vector field p on a $(2n + 2)$-dimensional conformal Lorentzian manifold M is said to be shear-free if its flow preserves the conformal Lorentz metric 'modulo p'. The $2n + 1$-dimensional orbit space N of the flow of p on M inherits then a CR-structure. For $n = 1$ this has been used by physicists to describe algebraically special solutions of the Einstein equation in terms of CR-manifolds. The correspondence between CR-manifolds and certain classes of Lorentzian manifolds works in both ways and has also been used by Hill, Lewandowski, and Nurowski to study the embedability problem of CR-manifolds in terms of conformal Lorentzian metrics.

In higher dimensions the related CR-manifolds possess an additional fibration structure, which makes their geometry more rigid. We study the properties of these structures and applications to higher-dimensional analogues of the Maxwell equation.

This is joint work with D. Alekseevsky and M. Ganji in progress.

Alexander Loboda *Voronezh State Technical University, Russia*
On moduli spaces and examples of holomorphically homogeneous hypersurfaces in $\mathbb{C}^3$

Abstract: The questions of the coefficient classification of holomorphically homogeneous strictly pseudo-convex real hypersurfaces in 3-dimensional complex spaces are discussed. The main attention is paid to the least studied class of surfaces, homogeneity of which is provided by Lie groups with the minimum possible dimensions, despite the 'increased' symmetry of polynomials of the 4th order from their normal Moser equations.

For all six potentially possible subfamilies of the studied class defined by Taylor coefficients of order 5, dimension estimates for moduli spaces of homogeneous surfaces are obtained. Examples are discussed of known affine homogeneous manifolds falling into the families under consideration.

Day II: Tuesday, September 12, 2017

Kang-Tae Kim *Pohang University of Science and Technology, South Korea*
On the generalization of Forelli's theorem

Abstract: The well-known Forelli theorem concerning the complex analyticity of functions was presented in 1977 by F. Forelli (and 1980 by W. Stoll). Then there were not much improvements until E. Chirka generalized it as part of his study of variations of Hartogs' analyticity theorem around 2005. We (Kim, Poletsky and Schmalz, and later with J. Joo) extended this study and obtained some further generalization. I would like to present the line of study by Kim-Poletsky-Schmalz and by Joo-Kim-Schmalz.

Stefan Nemirovsky *Steklov Mathematical Institute, Russia*
Continuation, approximation and topology in two complex variables

Abstract: The talk will be a survey of recent results and open problems on the topological properties of holomorphically, rationally, and polynomially convex domains in two complex dimensions.

Alexander Tumanov *University of Illinois at Urbana-Champaign, USA*
Symplectic non-squeezing in infinite dimension

Abstract: The celebrated Gromov's non-squeezing theorem of 1985 says that the unit ball in a symplectic space can be symplectically embedded in the circular cylinder only if the radius of the cylinder is at least 1. Hamiltonian differential equations provide examples of symplectic transformations in infinite dimension. Known results on the non-squeezing property in Hilbert spaces cover compact perturbations of linear symplectic transformations and several specific non-linear PDEs, including the periodic Korteweg - de Vries equation and the periodic cubic Schrödinger equation. We present a version of the non-squeezing theorem for Hilbert spaces. We apply the result to the discrete nonlinear Schrödinger equation. This work is joint with Alexander Sukhov.

Nadezhda Chuesheva *Kemerovo State University, Russia*
Some nonlinear equations of higher order

Abstract: After S.P. Novikov's article in 1974 the interest to the nonlinear Korteweg de Vries equation, Kadomtsev-Petviashvili equation and other nonlinear equations significantly grew. In these investigations exact solutions series of special nonlinear equations in partial derivatives play a big role.

I am going to talk about how to find some exact solutions for special series partial differential equations. Solution graphs of such problems for linear equations and for the KdV, Burgers-KdV, and Kadomtsev-Petviashvili equations are constructed.

Valery Beloshapka *Lomonosov Moscow State University, Russia*
Analytic complexity: 10 years later

Abstract: In this talk I will review my results in the field of the analytic complexity theory which were obtained over the last ten years. Among them a description of polynomials of complexity one, a stabilizer of a function theorem, and simple solutions of equations of mathematical physics. Then I will discuss several open problems.

Boris Bychkov *Higher School of Economics, Russia*
Degrees of the strata of Hurwitz spaces

Abstract: Let $\mathcal{H}_{0;k_1,\dots,k_m}$ be the space of meromorphic functions of degree $k_1+\ldots+k_m$ on genus 0 algebraic curve with the numbered multiplicities of the preimages $k_1,\dots,k_m$ of the point ∞ and the zero's sum of the finite critical values. The closure in $P\overline{\mathcal{H}}_{0;k_1,\dots,k_m}$ of the set of functions having prescribed ramifications forms the discriminant stratum. The degree of the stratum is the intersection index of its Poincaré dual class with the complementary degree of the first Chern class of the tautological line bundle. I will talk about the certain method of computation of the degrees of the strata of small codimension. As a consequence we will have a closed formulae for some series of so called double Hurwitz numbers and some new relations on the generating series for integrals of ψ-classes over the moduli space of stable genus 0 curves with marked points. My talk will follow the paper arXiv:1611.00504v1.

Alexander Shlapunov *Siberian Federal University, Russia*
Open mapping theorem for the Navier-Stokes equations in Hölder spaces

Abstract: We consider the Navier-Stokes equations in the layer $\mathbb{R}^n\times[0,T]$ over $\mathbb{R}^n$ with finite $T>0$ and $n\geq 2$. Using the standard fundamental solutions of the Laplace operator and the heat operator, we reduce the Navier-Stokes equations to a nonlinear Fredholm equation of the form $(I+K)u=f$, where K is a compact continuous operator in anisotropic normed Hölder spaces weighted at the point at infinity with respect to the space variables. Actually, the weight function is included to provide a finite energy estimate for solutions to the Navier-Stokes equations for all $t\in[0,T]$. On using the particular properties of the de Rham complex we conclude that the Fréchet derivative $(I+K)'$ is continuously invertible at each point of the Banach space under consideration and the map $I+K$ is open and injective in the space. In this way the Navier-Stokes equations prove to induce an open one-to-one mapping in the scale of Hölder spaces. On the last step, using weighted Energy type Estimates for $n=2$ we prove that the image of the map is closed, which gives a $2D$ Existence Theorem for the Navier-Stokes equations in the Hölder spaces weighted at the infinity with a finite time $T>0$. Finally, for data with a prescribed asymptotic behaviour at the infinity with respect to both the space and the time variables, we prove the existence of the smooth solution to the Navier-Stokes equations, possessing the Energy Estimates, over the space $\mathbb{R}^2\times(0,+\infty)$.

Gulmirza Khudayberganov *National University of Uzbekistan, Uzbekistan*
On integral formulas in the space of rectangular matrices

Abstract: The unit circle and its various multidimensional generalizations (a single n-dimensional ball, a polycircle, a matrix unit circle, four classical types of classes according to Cartan's classification, a matrix ball) are well-studied: to date, many important issues of several complex analysis have been solved, such as the description groups of automorphisms, obtaining integral formulas of Cauchy-Szegő, Bergman, Poisson type, proving necessary and sufficient conditions for the holomorphic continuability of functions from the boundary.

Day III: Wednesday, September 13, 2017

Timur Sadykov *Plekhanov Russian State University of Economics, Russia*
Amoeba-shaped polyhedral complex of an algebraic hypersurface

Abstract: Given a complex algebraic hypersurface H, we introduce a polyhedral complex which is a subset of the Newton polytope of the defining polynomial for H and enjoys the key topological and combinatorial properties of the amoeba of H. We provide an explicit formula for this polyhedral complex in the case when the hypersurface H is optimal.

Irina Antipova *Siberian Federal University, Russia*
Multidimensional Mellin transforms

Abstract: The Mellin transform figures prominently in complex analysis thanks to it being more adapted to using methods of the residue theory. A pair of convex domains $\Theta, U \subset \mathbb{R}^n$ encodes isomorphic functional spaces M_Θ^U, W_U^Θ which are transformed to each other by the direct and inverse Mellin transforms. Moreover, Θ and U predetermine the asymptotics of functions. In my talk I would like to focus on a subspace of rational functions in the space M_Θ^U which can be simply identified in case of quasi-elliptic denominators. The subspace is characterized by the fact that their denominators do not vanish in suitable toric compactifications of the space $\mathbb{R}^n$. It is important that any quasielliptic polynomial admits a power minorant x^u , $u \in U$, where U is its Newton polytope. I will speak about correspondence between individual terms in the asymptotic expansion of an original rational function and singularities of its Mellin transform. The proof is based on the representation of the integration orthant $\mathbb{R}^n_+$ in the form of a $(n-1)$-dimensional family of one-parameter curves and the application of the Leray multidimensional residue theory. The result was obtained in collaboration with A. Shchuplev and A. Tsikh.

Vladimir Kulikov *Siberian Federal University, Russia*
On convergence of multidimentional Mellin-Barnes integrals

Abstract: We obtain a criterion for the convergence of the Mellin-Barnes integral representing the solution to a general system of algebraic equations. This yields a criterion for a nonnegative matrix to have positive principal minors. The proof rests on the Nilsson-Passare-Tsikh Theorem about the convergence domain of the general Mellin-Barnes integral, as well as some theorem of a linear algebra on a subdivision of the real space into polyhedral cones.

Dmitry Pochekutov *Siberian Federal University, Russia*
Toric cycles in the complement of a complex curve in the 2-dimensional complex torus

Abstract: The amoeba of a complex curve in the 2-dimensional complex torus is its image under the projection onto the real subspace in the logarithmic scale. The complement to an amoeba is a disjoint union of connected components that are open and convex. A toric cycle is a 2-cycle in the complement to a curve associated with a component of the complement to an amoeba. We prove homological independence of toric cycles in the complement to a complex algebraic curve with amoeba of maximal area.

Day IV: Thursday, September 14, 2017

Shinichi Tajima *Tsukuba University, Japan*
A new effective algorithm for computing Grothendieck local residues

Abstract: Grothendieck local residue is considered in the context of complex analysis. A new effective method is described for computing Grothendieck local residues. The keys of our new approach are the use of the concept of local cohomology and the Grothendieck local duality. A new efficient procedure is used for solving an extended ideal membership problem in local rings.

Alain Yger *Université de Bordeaux, France*
Averaging procedures versus effectivity questions in the arithmetic setting

Abstract: Is is now a well established fact (since the long term work of M. Andersson, H. Samuelsson-Kalm, E. Wulcan) that Bochner-Martinelli type currents play a central role with respect to effectivity questions in division or interpolation questions in complex analytic geometry. Unfortunately, the fact that the construction of such objects relies deeply on an averaging process of the Fubini-Study type prevents them (at least for the moment) to play such a proeminent role when dealing over an arithmetic context (for example when such division or interpolation questions are settled with $\mathbb{Q}$ or a number field as base field). Nevertheless, some concepts involving such an averaging process provide key indicators for arithmetic complexity : such is the case for the Mahler measure or for the so-called Ronkin function attached to polynomials defined over such a field. I will present and discuss in this lecture some recent results by Farhad Babaee and June Huh, Roberto Gualdi, Ibrahima Hamidine in order to suggest from such discussion how the averaging process which is inherent to Bochner-Martinelli construction could be transposed from the archimedean to ultrametric context in order to fit with effectivity questions when the basis field is $\mathbb{Q}$ or a number field.

Gabino González-Diez *Universidad Autónoma de Madrid, Spain*
Universal covers of complex surfaces and arithmeticity

Abstract: A consequence of the results of Bers and Griffiths on the uniformization of complex algebraic varieties is that a complex projective surface S admits a Zariski open set, which can be viewed as a family of Riemann surfaces of finite type, such that its universal cover is a contractible bounded domain $\mathscr{B}$ of $\mathbb{C}^2$.

In this talk I will attempt to show that these bounded domains can be realized as graphs of holomorphic motions of the unit disk, to describe which holomorphic motions give rise to such uniformizing domains $\mathscr{B}$ and to characterize which among them correspond to complex surfaces S defined over a number field. As a side product these results will allow us to construct explicit families of pairwise non-isomorphic contractible bounded domains of $\mathbb{C}^2$.

The talk will based on joint work with Sebastián Reyes-Carocca.

Yury Eliyashev *Siberian Federal University, Russia*
Supercurrents and generalized amoebas

Abstract: Recently, Igor Krichever proposed a generalization of the amoeba of a complex planar curve. We proposed a concept of multidimensional generalized amoeba and proved that the generalized amoebas behave similarly to the classical amoebas. To deal with the

generalized amoeba the language of tropical superforms and supercurrents turn out to be useful. This language was developed by Aron Lagerberg and proved to be useful in tropical geometry. In our talk we will report main facts about classical and generalized amoebas and explain how supercurrents can be applied to study geometry of amoebas.

Alain Hénaut *CNRS & Université de Bordeaux, France*
Implicit differential equations and planar web geometry

Abstract: In the complex setting, let $F(x, y, y') = 0$ be an analytic or an algebraic differential equation with y'-degree d. We deal with the qualitative study of such equations through the geometry of the planar d-web generated by the generic family of integral curves. Invariants of these configurations as abelian relations (closely related to Abel's addition theorem) or infinitesimal symmetries are discussed, in the nonsingular case and through the singularities and their residues as well. Basic examples as models are given from different domains including classic algebraic geometry and Frobenius 3-manifolds, that is WDVV-equations. By using meromorphic connections methods 'à la Cartan-Spencer' for $d \geq 3$ or 'à la Chern' for $d = 3$, new results are presented.

Sergey Smirnov *Lomonosov Moscow State University, Russia*
Darboux integrable systems

Abstract: The concept of integrability in the theory of integrable systems can be understood in many different ways depending on the class of equations that is being considered. For example, the notion of *Liouville integrability* is applied to finite-dimensional systems, and evolution equations in two independent variables are usually integrated *by means of inverse scattering theory*. I'll talk about *Darboux integrable systems*, that is, a class of hyperbolic systems that can be integrated more or less explicitly. The most simple example of such systems is the classical *Liouville equation*

$$u_{xy} = e^u.$$

Its general solution was been obtained by J. Liouville in 1853. A less trivial example of Darboux integrable system is given by *two-dimensional Toda lattices*. These systems have first appeared in differential geometry in the end of the 19th century and almost a century later they have reappeared in theoretical physics.

The notion of Darboux integrability can be easily extended to the case of differential-difference or entirely difference hyperbolic equations. I'll talk about one idea that allows to obtain characteristic integrals for some of the Toda lattices and for their discrete analogs in order to prove their Darboux integrability.

Sachiko Hamano *Osaka City University, Japan*
Variational formulas for hydrodynamic differentials and pseudoconvexity

Abstract: We establish the variational formulas of hydrodynamic differentials for the deforming open Riemann surface $R(t)$ of genus one with complex parameter t, and give some applications to the rigidity theorems under pseudoconvexity.

Day V: Friday, September 15, 2017

Azimbay Sadullaev *National University of Uzbekistan, Uzbekistan*

Further developments of the pluripotential theory

Abstract: In the talk we give an overview of the development of the pluripotential theory based on plurisubharmonic functions and the Monge-Ampere operator. The most importantly, the pluripotential theory have been used successfully in solving various problems, that have accumulated in multidimensional complex analysis and in the theory of plurisubharmonic functions.

From the 1990s there were attempts to develop, expand the theory to a wider classes, in particular to a class of m-subharmonic (m-sh) functions ($1 \leq m \leq n$): upper semicontinuous in a domain $D \subset \mathbb{C}^n$ is said to be m-subharmonic in D, $u \in m\text{-}sh(D)$ if $\mathrm{dd^c}u \wedge \beta^{m-1} \geq 0$ as current, where $\beta = \mathrm{dd^c}|z|^2$ is the standard volume form in the space $\mathbb{C}^n$.

Note that $psh(D) = 1\text{-}sh(D) \subset m\text{-}sh(D) \subset n\text{-}sh(D) = sh(D)$. Such functions have an excellent geometric characteristic and they have been studied in the works of Z. Khusanov, B. Abdullaev, L.N. Ho, M.Verbitsky, D.Jouce, B. Drnovšek and F. Forstnerič, et al. In a series of works of F.R. Harvey and H. B. J. Lawson class of such functions is applied in calibrated geometries.

Georgy Shabat *Russian State University for the Humanities & Lomonosov Moscow State University, Russia*

Meromorphic functions on Riemann surfaces with no more than 4 critical values

Abstract: We consider smooth compact connected Riemann surfaces and the meromorphic functions on them that can be looked at as the coverings of the Riemann sphere with the branching set, consisting of 3 or 4 points (the smaller number of branching points is trivial or impossible). The coverings with 3 branching points, also called the Belyi ones, are rigid; their combinatorial description in terms of certain graphs on surfaces (called dessins d'enfants) is known. The coverings with 4 branching points admit 1-parameter deformations, locally depending on the cross-ratios of the branching points; their combinatorial description also exists, but is more involved and not yet standartized.

The sets of coverings with 4 branching points constitute nets of complex 1-dimensional subsets in the Hurwitz, Teichmüller or moduli spaces; these nets are the main objects of our interest. Some examples will be presented and the problems formulated; the questions concerning certain extremal quasiconformal maps will be asked.

Alexander Komlov *Steklov Mathematical Institute, Russia*

Hermite–Padé approximants for meromorphic functions on a compact Riemann surface

Abstract: Let $\mathfrak{R}$ be a compact Riemann surface and $\pi : \mathfrak{R} \to \widehat{\mathbb{C}}$ be an $(m+1)$-fold branched covering of the Riemann sphere $\widehat{\mathbb{C}}$, $m \geqslant 1$. Let $f_1, f_2, \ldots, f_m$ be meromorphic functions on $\mathfrak{R}$, and let the functions $1, f_1, f_2, \ldots, f_m$ be independent over the field $\mathbb{C}(z)$ of rational functions. Let $\circ$ be an arbitrary point of $\mathfrak{R}$ that is not critical for the projection π. Without loss of generality we suppose that $\circ \in \pi^{-1}(\infty)$ and denote $\boldsymbol{\infty}^{(0)} := \circ$. In a neighbourhood of ∞ we set $f_{j,\infty}(z) := f_j(\pi_0^{-1}(z))$, where π_0 is the biholomorphic restriction of π to a neighbourhood of $\boldsymbol{\infty}^{(0)}$. For convenience we suppose that the germs $f_{j,\infty}(z)$ are holomorphic at ∞.

Let us define the *Hermite–Padé polynomials of the first kind* $Q_{n,0}, \ldots, Q_{n,m}$ of order $n \in \mathbb{N}$ for the tuple of germs $[1, f_{1,\infty}, \ldots, f_{m,\infty}]$ at the point $\infty \in \widehat{\mathbb{C}}$ in the following way: $\deg Q_{n,j} \leqslant n$, $j = 0, \ldots, m$, at least one of $Q_{n,j} \not\equiv 0$, and the following asymptotic relation

at ∞ holds true:

$$Q_{n,0}(z) + \sum_{j=1}^{m} f_{j,\infty}(z) Q_{n,j}(z) = O\left(\frac{1}{z^{m(n+1)}}\right) \quad \text{as } z \to \infty. \tag{1}$$

The talk is devoted to the study of limiting zero distribution of such Hermite–Padé polynomials $Q_{n,j}(z)$ and the study of asymptotic behaviour of the quotients $\frac{Q_{n,j}(z)}{Q_{n,k}(z)}$, $j,k = 0,\dots,m$. Our research uses Nuttall's approach that is based on the special 'Nuttall's partition' of the Riemann surface $\mathfrak{R}$ into sheets.

The talk is based on the joint work with E.M.Chirka, R.V.Palvelev, and S.P.Suetin, Russ.Math.Surv. 72:4 (2017). The work was also discussed with N.G.Kruzhilin.

Alekos Vidras *University of Cyprus, Cyprus*

Cauchy-Fantappiè integral representation formula for class of holomorphic functions in tubes

Abstract: Let $T_{B_1} = \mathbb{R}^2 \times i\{(y_1,y_2) \in \mathbb{R}^2 : y_1^2 + y_2^2 < 1\}$ be the (unbounded) tube defined by the function

$$\Phi_1(\zeta,\bar{\zeta}) = \left(\frac{\zeta_1 - \bar{\zeta}_1}{2i}\right)^2 + \left(\frac{\zeta_2 - \bar{\zeta}_2}{2i}\right)^2 - 1.$$

Let $H^2(T_{B_1})$ be the Hardy space of holomorphic functions $f(z) = \int\limits_{\mathbb{R}^2} h(t) e^{2\pi i z\cdot t} dt$, $z \in T_{B_1}$, where f satisfies

$$\sup_{y \in B_1} \int\limits_{\mathbb{R}^2} |f(t)|^2 e^{-4\pi y\cdot t} dt \le A^2 < +\infty.$$

We prove that every function $f \in H^2(T_{B_1})$ is represented by a Cauchy-Fantappiè formula supported on the boundary ∂T_{B_1}. Namely, for every $z \in T_{B_1}$ one has

$$f(z) = \int\limits_{\mathbb{R}^2 \times iS^1} \frac{f(\zeta)\partial\Phi_1(\zeta,\bar{\zeta}) \wedge \bar{\partial}\partial\Phi_1(\zeta,\bar{\zeta})}{< \nabla\Phi_1(\zeta,\bar{\zeta}), \zeta - z >^2}.$$

Victor Chueshev *Kemerovo State University, Russia*

Prym differentials with matrix characters on finite Riemann surfaces

Abstract: Theory of multiplicative functions and Prym differentials for special (scalar) characters on compact Riemann surfaces found a lot of applications in theory of functions, in analytic number theory and equations of mathematical physics.

We prove existence of a matrix multiplicative functions and Prym $m-$differentials for every matrix characters with values in $GL(n,\mathbb{C})$ on finite Riemann surfaces and that such differentials holomorphically depend on matrix characters.

Armen Sergeev *Steklov Mathematical Institute, Russia*

Adiabatic limit in Seiberg–Witten equations

Abstract: Seiberg–Witten equations, along with Yang–Mills equations, are the limiting cases of a more general supersymmetric Yang–Mills theory. But, opposite to the conformally invariant Yang–Mills equations, the Seiberg–Witten equations are not invariant under the change of scale. So to draw a useful information from these equations one should introduce the scale parameter λ into them and take the limit $\lambda \to \infty$. This is the adiabatic limit and we study the adiabatic equations obtained from Seiberg–Witten equations in this limit. It turns out that the arising adiabatic equations on a 4-dimensional symplectic manifold may be identified with a non-linear $\bar{\partial}$-equation on a pseudoholomorphic curve.